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TECHNICAL NOTES

INFLUENCE OF BUOYANCY ON THE VERTICAL FLOW AND HEAT TRANSFER IN A SHROUDED FIN ARRAY

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NOMENCLATURE

C	clearance parameter, c/H
c	tip clearance, Fig. 1
D_h	hydraulic diameter, $2s(H+c)/(H+s)$
f	friction factor, $-2D_h(\rho g+dp/dz)/(\rho\bar{w}^2)$
$(f\,Re)_0$	the value of $f\,Re$ at $Ra = 0$
g	acceleration due to gravity
H	fin height, Fig. 1
h	overall heat transfer coefficient, $q/(T_w - T_b)$
k	thermal conductivity of the fluid
Nu	Nusselt number, hH/k
Nu_0	the value of Nu at $Ra = 0$
p	pressure
Q	heat input to the calculation domain per unit axial length
q	average heat flux, $Q/(H+s/2)$
Ra	Rayleigh number, equation (5)
Re	Reynolds number, $\rho\bar{w}D_h/\mu$
S	dimensionless spacing, s/H
s	spacing between fins, Fig. 1
T	fluid temperature
T_b	bulk temperature, $\iint wT\,dx\,dy/\iint w\,dx\,dy$
T_w	temperature of the base and fins
w	velocity in the z direction
$\bar{w}$	average value of w, $\iint w\,dx\,dy/\iint dx\,dy$
X, Y	dimensionless coordinates, equation (3)
x, y	cross-sectional coordinates
z	coordinate in the vertical upward direction.

Greek symbols

α	thermal diffusivity of the fluid
β	coefficient of volumetric expansion
θ	dimensionless temperature, equation (6)
μ	viscosity of the fluid
ρ	reference density of the fluid
ϕ	the variable $\theta\Omega^*$, equation (6)
Ω	dimensionless axial velocity, equation (4)
Ω^*	the integral $\iint\Omega\,dX\,dY$ over the calculation domain.

INTRODUCTION

THE SHROUDED fin array, schematically shown in Fig. 1, represents a situation in which the heat transfer from the fins is dependent on the distribution of the flow in the duct cross section. The amount of clearance between the fin tip and the shroud has been shown in ref. [1] to have a significant effect on the heat transfer. Buoyancy forces have a further effect on the flow field and the associated heat transfer. This effect for a 'horizontal' shrouded fin channel was investigated in ref. [2]. The purpose of this note is to consider the combined forced and free convection in a 'vertical' channel shown in Fig. 1. A similar investigation for circular tubes with radial internal fins has been reported in ref. [3].

When the clearance is present, the fluid prefers to flow in the clearance space rather than in the inter-fin space, thus reducing the heat transfer from the fins. However, the buoyancy effects can reverse this trend under suitable conditions. If the fins are hotter than the fluid, the buoyancy force assists the upflow near the fins and opposes it in the clearance space. Further, a heated upflow is identical to a cooled downflow.

As explained in ref. [3], the case of cooled upflow or heated downflow is not of much practical interest since it leads to the so-called 'run-away' solutions. Therefore, only the case of heated upflow (or cooled downflow) is considered here. For this case, the retardation of the flow in the clearance region tends to cause reverse flow at a critical value of the Rayleigh number. The solutions presented here do not include the range beyond the critical Rayleigh number.

The formulation of the problem and the solution technique employed here closely resemble those in ref. [3]; therefore, only the important details are given.

ANALYSIS

The flow is considered to be fully developed under the thermal boundary condition of uniform heat input per unit axial length. Further, the base and fins are supposed to be made of a material of high thermal conductivity so that they assume a uniform temperature over any cross section. The fin thickness t is assumed to be negligible in comparison with the inter-fin spacing s. The flow is regarded as laminar and the fluid properties constant except in the buoyancy term. In the fully developed region, the axial velocity is independent of z, while the other velocity components are zero. Further, all the temperatures rise linearly with z at the same rate.

With these assumptions, the governing differential equations for w and T can be cast into the following dimensionless form:

$$\frac{\partial^2\Omega}{\partial X^2}+\frac{\partial^2\Omega}{\partial Y^2}+Ra\,\phi+1=0, \tag{1}$$

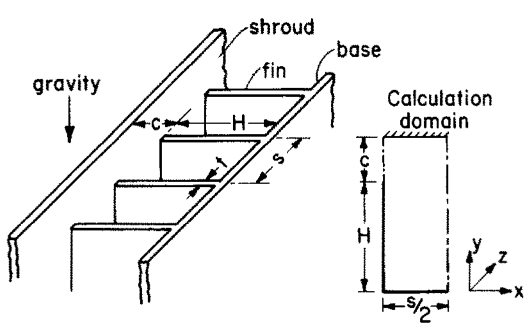

FIG. 1. The vertical shrouded fin array.

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Table 1. Friction factor and heat transfer results

Ra	fRe	Nu	Ra	fRe	Nu
$S = 0.5$,	$C = 0$		$S = 0.1$,	$C = 0$	
0	61.7	6.78	0	84.0	38.6
10	62.0	6.78	10^3	84.0	38.6
10^2	64.3	6.81	10^4	84.8	38.6
10^3	87.4	7.04	10^5	92.1	38.8
10^4	290	8.71	10^6	164	40.4
9.77×10^4 †	1540	15.6	6.44×10^7 †	3280	83.0
$S = 0.5$,	$C = 0.25$		$S = 0.1$,	$C = 0.25$	
0	80.6	6.26	0	49.9	1.03
10	81.1	6.27	10	49.9	1.03
10^2	86.1	6.37	10^2	50.0	1.05
10^3	133	7.01	10^3	55.5	1.16
10^4	513	9.11	10^4	87.8	2.52
1.7×10^4 †	766	10.1	6.35×10^4 †	138	14.2
$S = 0.5$,	$C = 0.5$		$S = 0.1$,	$C = 0.5$	
0	76.1	2.39	0	17.3	0.284
10	78.6	2.45	10	17.7	0.285
10^2	98.5	2.99	10^2	21.0	0.297
10^3	215	6.02	10^3	48.0	0.419
2.2×10^3 †	322	7.19	5.13×10^3 †	112	1.09
$S = 0.5$,	$C = 1.0$		$S = 0.1$,	$C = 1.0$	
0	50.8	0.659	0	5.86	0.128
10	71.0	0.748	10	7.79	0.132
10^2	185	1.62	10^2	23.2	0.163
2.2×10^2 †	264	2.73	3.58×10^2 †	55.9	0.237

† Critical Rayleigh number.

$$\frac{\partial^2 \phi}{\partial X^2} + \frac{\partial^2 \phi}{\partial Y^2} - \Omega = 0, \quad (2)$$

where the dimensionless quantities have the meaning

$$X = x/H, \quad Y = y/H, \quad (3)$$

$$\Omega = -w\mu/[H^2(\rho g + \mathrm{d}p/\mathrm{d}z)], \quad (4)$$

$$Ra = \rho\beta g H^4(\mathrm{d}T_b/\mathrm{d}z)/(\alpha\mu), \quad (5)$$

$$\theta = (T - T_w)/(Q/k), \quad \Omega^* = \iint \Omega \, \mathrm{d}X \, \mathrm{d}Y, \quad \phi = \theta\Omega^*. \quad (6)$$

Here Q is the heat input per unit axial length for the calculation domain shown in Fig. 1; the integral in equation (6) is also to be taken over this calculation domain. The boundary conditions are given by:

base and fins: $\Omega = 0$, $\phi = 0$;

shroud: $\Omega = 0$, $\partial\phi/\partial Y = 0$;

remaining boundaries: $\partial\Omega/\partial X = 0$, $\partial\phi/\partial X = 0$.

Equations (1) and (2) represent a system of coupled Poisson equations, which were solved by a finite-difference procedure. The coupling between the two equations was handled by the special line-by-line method developed in ref. [3], which involves the simultaneous solution for the two variables along a line. Also, the coupled block-correction procedure was employed [3]. Whereas the boundary conditions in ref. [3] for

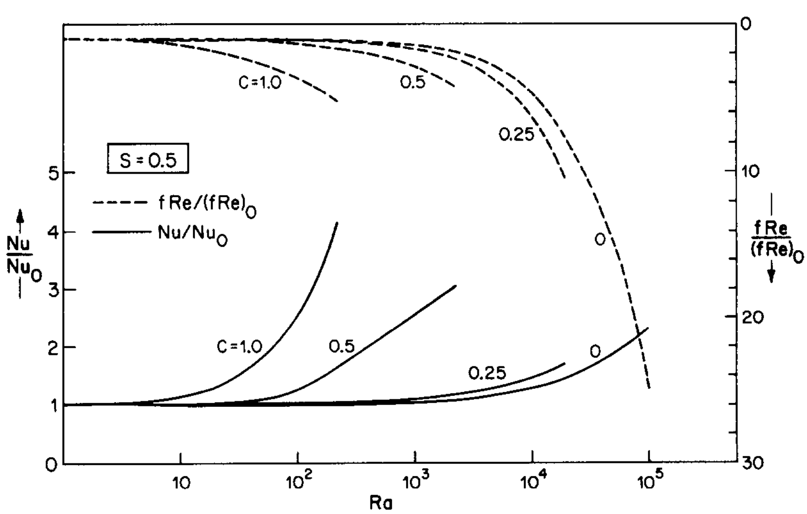

FIG. 2. Effect of buoyancy on friction and heat transfer for $S = 0.5$.

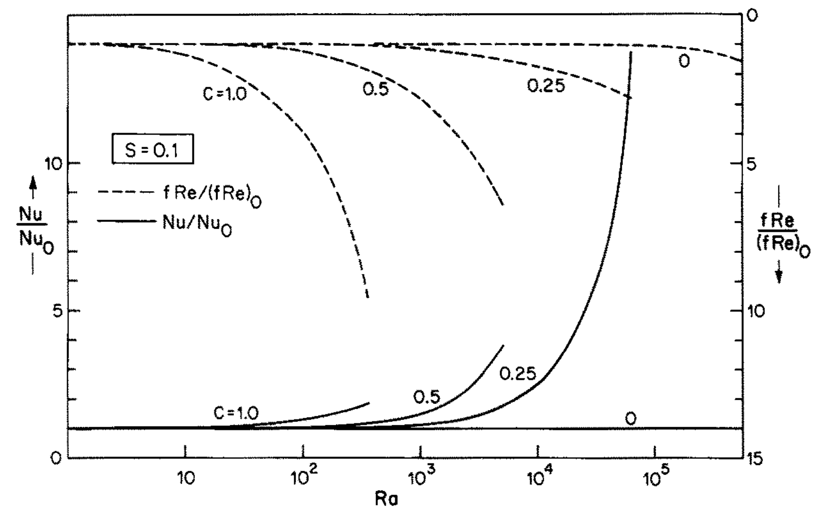

FIG. 3. Effect of buoyancy on friction and heat transfer for $S = 0.1$.

Ω and ϕ were identical, the present problem involves dissimilar conditions at the shroud. This leads to a slight difference in the form of the finite-difference equations and in the method of solution. However, these departures from ref. [3] are sufficiently straightforward, and no detailed description seems necessary. It may be noted that, in the present work, the block correction as well as the line-by-line procedure was employed both along the x- and y-direction lines.

RESULTS

All the results reported here pertain to a 14×50 grid in the xy coordinates. The grid spacing was nonuniform; a finer grid was used near the base, fin, and shroud surfaces. Trial calculations on different grids indicate that the results reported here are accurate to at least 0.5%. Also, the results obtained for $Ra = 0$ agree, wherever applicable, with the corresponding results in ref. [1]. The number of iterations required to attain convergence to five significant figures was about 15 for the cases of $S = 0.1$ and about 25 for $S = 0.5$. The faster convergence for $S = 0.1$ results from the fact that the known values of Ω and ϕ on the fins have greater control over the solution when the fins are closely spaced.

The computations were performed for $S = 0.1$ and 0.5, for $C = 0, 0.25, 0.5$, and 1, and Rayleigh numbers up to the critical value for each case.

Overall results

The computed values of fRe and Nu are listed in Table 1. The influence of buoyancy can be best seen from Figs. 2 and 3 where the ratios $fRe/(fRe)_0$ and Nu/Nu_0 are plotted. These ratios are significantly different from unity, and in general, the values of $fRe/(fRe)_0$ are greater than those of Nu/Nu_0. For a given Rayleigh number, the ratio Nu/Nu_0 increases with the clearance C. This can be understood by noting that the large-C cases allow a greater nonuniformity of temperature over the cross section; hence the buoyancy effect is stronger. For the same reason, the value of the critical Rayleigh number decreases with increasing clearance. The highly nonuniform temperature field in the large-C cases makes it more likely that the flow will reverse somewhere in the cross section.

Velocity and temperature distributions

The increased friction and heat transfer for the buoyancy-affected situation results from the redistribution of the flow and the associated change in the temperature field. Some idea

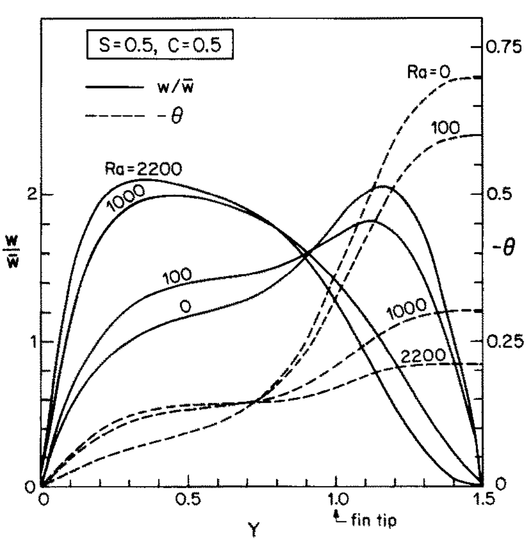

FIG. 4. Velocity and temperature profiles for $S = 0.5$.

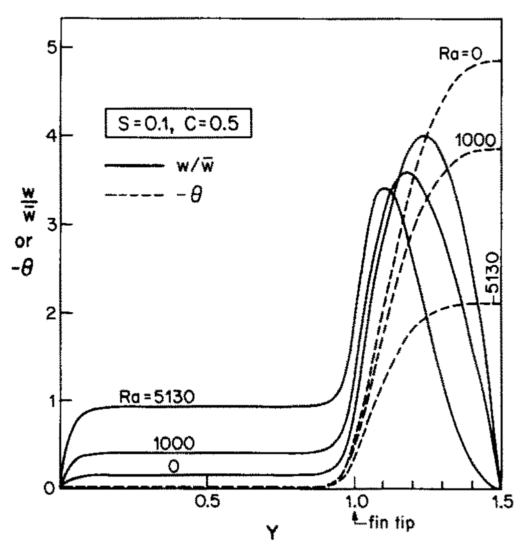

FIG. 5. Velocity and temperature profiles for $S = 0.1$.

of the velocity and temperature distributions can be obtained from the profiles plotted along the symmetry line between two adjacent fins. Figures 4 and 5 present such profiles for a few typical cases. As the Rayleigh number increases, the velocity and temperature distributions tend to become more uniform, thus increasing the gradients at the base surface. Also, it is interesting to note in Fig. 5 that the closely spaced fins ($S = 0.1$) exert a strong influence throughout the inter-fin space so that the values of $w/\bar{w}$ and θ remain rather small and uniform for $Y < 1$.

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TURBULENT HEAT AND MASS TRANSFER IN NEWTONIAN AND DILUTE POLYMER SOLUTIONS FLOWING THROUGH ROUGH PIPES

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NOMENCLATURE

- d pipe diameter
- e roughness height
- f friction factor
- f_s friction factor for a smooth pipe
- Nu Nusselt number
- Pr Prandtl number
- Re Reynolds number
- Sc Schmidt number
- Sh Sherwood number
- T time period for sublayer development
- T^+ dimensionless time period, Tv_0^2/ν
- v_0 friction velocity.

Greek symbol
- ν kinematic viscosity.

INTRODUCTION

THE CONCEPT of an eddy diffusivity has been widely used to discuss the increase in heat and mass transfer owing to the surface roughness as well as the studies of transport processes near smooth surfaces [1–6]. On the other hand, the concept of a surface renewal, which has been successfully used to predict turbulent heat and mass transfer rates from smooth surfaces to flowing fluids [7], has been scarcely applied. Hughmark [8] utilized the penetration model to rough surface pipes. In Hughmark's work, the influence of surface roughness on transport processes is taken account of only in the turbulent core region. Furthermore, the constants in the expressions for the mass transfer coefficients in the laminar sublayer and the transition regions were determined by comparison with heat and mass transfer data in smooth pipes.

In this work, the applicability of the surface renewal concept is discussed. The proposed model based on the periodic transitional sublayer model (see ref. [9]) is compared with the available experimental data on turbulent heat and mass transfer in a rough pipe. The influence of polymer additives is also discussed.

SURFACE RENEWAL MODEL

It has been shown [9, 10] that the periodic transitional sublayer model proposed by Pinczewski and Sideman [9] provides a satisfactory representation of turbulent heat and mass transfer in Newtonian and non-Newtonian fluids flowing through smooth pipes. We discuss the effect of surface roughness on the transfer rates using the periodic transitional sublayer model.

When the Schmidt number is sufficiently high, the rate of mass transfer in a smooth pipe is determined by the rate of renewal of the thin wall-layer. The roughness causes disturbances in the viscous sublayer which penetrate to the wall and promote turbulent transport. Therefore, the thickness of the viscous sublayer and of the wall-layer decreases with increasing roughness. As the result, even at high Schmidt numbers, the thickness of the concentration boundary layer might be larger than that of the wall-layer. In other words, the effect of the thin wall-layer is negligible and the resistance to mass transfer is mainly due to the periodically developing sublayer flow. For $Sc > 1$, the resistance of the turbulent core is small in comparison to that within the sublayer and can be assumed to be negligible. This situation for mass transfer from a rough surface at high Schmidt numbers is corresponding to the mass transfer in a smooth pipe for $Sc = O(1)$. The validity of this assumption is verified *a postiori* by a comparison of the predictions of the model with experimental data.

An expression for the Sherwood number for Newtonian fluids and dilute polymer solutions under the maximum drag reduction condition for $Sc = O(1)$ may be written as [9, 10]

$$Sh = \frac{1}{T^+}\sqrt{\left(\frac{f}{2}\right)}\,Re\,Sc^{1/2}\,(1.11+0.44\,Sc^{-1/3}-0.70\,Sc^{-1/6}). \tag{1}$$

As described above, this equation might be applicable to discuss turbulent mass transfer in a rough pipe at fairly large Schmidt numbers.

Because of the lack of pertinent information, the